Period 1, April 23, 2025

If $x + 3y^{\frac{1}{3}} = y$, what is $\frac{dy}{dx}$ at the point $(2, 8)$?

$$1 + 3 \cdot \frac{1}{3} y^{\frac{1}{3}-1} \frac{dy}{dx} = \frac{dy}{dx}$$

$$1 + y^{-\frac{2}{3}} \frac{dy}{dx} = \frac{dy}{dx}$$

$$1 + \frac{1}{\sqrt[3]{y^2}} \frac{dy}{dx} = \frac{dy}{dx}$$

$$1 + \frac{1}{\sqrt[3]{8^2}} \frac{dy}{dx} = \frac{dy}{dx}$$

$$1 + \frac{1}{\sqrt[3]{64}}$$

$$1 + \frac{1}{4} \frac{dx}{dx} = 1\frac{dy}{dx} - \frac{1}{4}\frac{dy}{dx}$$

$$-\frac{1}{4}\frac{dy}{dx}$$

$$\frac{4}{3} \cdot 1 = \frac{4}{3} \cdot \frac{3}{4} \frac{dy}{dx}$$

$$\frac{4}{3} = \frac{dy}{dx}$$

(A) $\frac{1}{3}$

(B) $\frac{3}{4}$

(C) $\frac{5}{4}$

(D) $\frac{4}{3}$

$\lim\limits_{x \to \infty} \frac{10 - 6x^2}{5 + 3e^x}$ is $= \frac{0 - 12x}{0 + 3e^x} \Rightarrow \frac{-12}{3e^x} = \frac{-12}{\infty} = 0$

(A) -2

(B) 0

(C) 2

(D) nonexistent

$$f(x) = \begin{cases} -x^2 + 3 & \text{if } x \le 5 \\ -10x + 28 & \text{if } x > 5 \end{cases}$$

$-(5)^2+3 = -25+3 = -22$
$-10(5)+28 = -50+28 = -22$ } continious

Let f be the function defined above.

($-2x$ } $-2 \cdot 5 = -10$) diff
-10 } $-10 = -10$ }

(A) f is continuous and differentiable at $x = 5$.

(B) f is continuous but not differentiable at $x = 5$.

(C) ~~f is differentiable but not continuous at $x = 5$.~~

(D) f is defined but neither continuous nor differentiable at $x = 5$.

If $\frac{dy}{dx} = 2 - y$, and if $y = 1$ when $x = 1$, then $y =$

$\frac{dx}{2-y} \cdot \frac{dy}{dx} = (2-y) \cdot dx$
$\frac{(2-y)}{2-y}$

(A) $2 - e^{x-1} = y$

(B) $2 - e^{1-x} = y$

(C) ~~$2 - e^{-x}$~~ $= 1$

(D) ~~$2 + e^{-x}$~~

$\int \frac{1}{2-y} dy = \int dx$

$u = 2-y$

$\ln_e |2-y| = -x+1$

$-\ln|2-y| = x + C$

$\ln|2-y| = -x + c_1$

$e^{-x+1} = 2-y$
$-2 \qquad -2$

$\ln(2-1) = -1 + c_1$

$\ln 1 = 0 = -1 + c_1$

$c_1 = 1$

$-1 \cdot (-2 + e^{1-x}) = (-y) \cdot -1$

A particle moves along the y-axis so that at time $t \geq 0$ its position is given by $y(t) = t^3 - 4t^2 + 4t + 3$. Which of the following statements describes the motion of the particle at time $t = 1$?

- Ⓐ The particle is moving down the y-axis with decreasing velocity. ✔ 9
- Ⓑ The particle is moving down the y-axis with increasing velocity. 5
- Ⓒ The particle is moving up the y-axis with decreasing velocity. 6
- Ⓓ The particle is moving up the y-axis with increasing velocity. 3

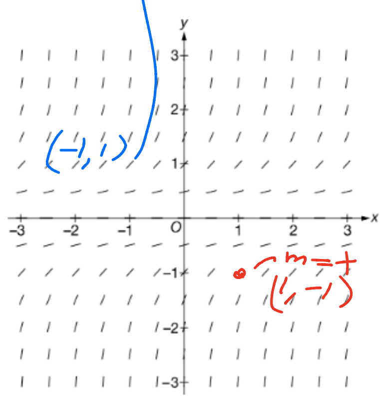
$V(T) = 3T^2 - 8T + 4$

$V(1) = 3(1)^2 - 8(1) + 4 = -1$] down

$a(T) = 6T - 8$

$a(1) = 6(1) - 8 = -2$] change in velocity

$m = +$



Shown is a slope field for which of the following differential equations?

- Ⓐ $\cancel{\frac{dy}{dx} = |x+y|}$ $\quad \frac{dy}{dx} = |-1+1| = 0$ 2
- Ⓑ $\cancel{\frac{dy}{dx} = x^3}$ $\quad \frac{dy}{dx} = (-1)^3 = -1$ 3
- Ⓒ $\cancel{\frac{dy}{dx} = y^3}$ $\quad \frac{dy}{dx} = (1)^3 = 1 \quad (-1)^3 = -1$ 6
- Ⓓ $\frac{dy}{dx} = y^2$ $\quad \frac{dy}{dx} = (1)^2 = 1 \quad (-1)^2 = +1$ ✔ 11

$y+5=2e^{6x}$

The equation $y = 2e^{6x} - 5$ is a particular solution to which of the following differential equations?

(A)	$y' - 6y - 30 = 0$	✓ 5
(B)	$2y' - 12y + 5 = 0$	8
(C)	$y'' - 5y' - 6y = 0$	6
(D)	$y'' - 2y' + y + 5 = 0$	4

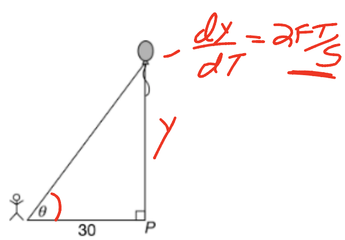
$y = 2e^{6x} - 5$

$y' = 2 \cdot e^{6x} \cdot 6 - 0$

$y' = 2e^{6x} \cdot 6$

$y' = 6(y+5) = 6y + 30$

$y' - 6y - 30 = 0$



A person stands 30 feet from point P and watches a balloon rise vertically from the point, as shown in the figure above. The balloon is rising at a constant rate of 2 feet per second.

What is the rate of change, in radians per second, of angle θ at the instant when the balloon is 40 feet above point P?

$\tan\theta = \frac{y}{30}$

$30\tan\theta = y$

$30\sec^2\theta \cdot \frac{d\theta}{dT} = \frac{dy}{dT}$

(A)	$\frac{3}{100}$	2
(B)	$\frac{3}{125}$	✓ 10
(C)	$\frac{1}{12}$	6
(D)	$\frac{5}{27}$	6